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Original Article

Estimating The Risk Function in Light of The Pressures That Kidney Failure Patients are Exposed

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Abstract
This article sheds light on the mixed survival distribution, emphasizing its potential for describing the survival behavior of kidney failure patients under the stressors they encounter in their daily lives. The analysis focused on the mixed hazard function derived from the mixed survival distribution, with the parameters of the distribution and its associated functions estimated to better understand the behavior of the hazard function over time. A goodness-of-fit test was applied to evaluate how well the data fit the mixed distribution, while comparison criteria were employed to identify the most suitable distribution compared to classical models. The mixed hazard function was further compared with the Weibull hazard function. To illustrate its applicability, real data were used, representing the survival times to death for 153 patients at the Kidney Services Center in Al-Khums. The results revealed that the mixed distribution provided an excellent fit to the real data and outperformed the Weibull distribution in representing survival behavior. Moreover, the analysis showed that the hazard function increases over time, indicating a higher probability of death as the disease progresses. The hazard function curves for kidney patients at different stress levels (10%–99%) demonstrated that the probability of being at risk rises with time. At low stress levels (10%), the increase is gradual, whereas at high stress levels (95%), the risk grows substantially. This finding highlights stress as a significant factor that accelerates the rate of risk in kidney patients. Similarly, the cumulative hazard function across different survival periods showed that the accumulated probability of risk exposure increases with survival duration. The cumulative hazard function rises more rapidly at higher stress levels, while the increase is slower at lower stress levels. This indicates that the intensity of stress plays a clear role in accelerating the accumulation of risk among kidney patients.
Keywords. Comparison Criteria, Goodness-of-Fit Test, Kidney Patients, Mixed Survival Distribution, Mixed Hazard Function.

Introduction
Kidney failure is the term used to describe the end-stage kidney disease, resulting from one or both kidneys failing or failing to perform their function effectively, which is to remove body waste and excess fluid. A healthy kidney removes waste and fluid from the bloodstream and excretes it in the urine [8]. The quality of the procedures used in statistical analysis depends largely on the assumed model or probability distribution. However, many problems remain, such as the fact that the distribution of real-world data does not follow any of the classical probability models. Therefore, several researchers have extensively and repeatedly discussed distributions in experimental statistical data to select the appropriate model and related issues in applied sciences such as the environment, medicine, engineering, modelling, and analysis of experimental data, but none of the studies have addressed the article of mixed hazard function in the presence of stressors that kidney failure patients are exposed to in their daily lives.
It is well known that a kidney failure patient is exposed to stress during their daily lives due to the medication they take and the dialysis they undergo, all of which can impact their daily lifestyle. If we assume that P represents the probability that a kidney failure patient is able to handle such stressors so that they do not affect their daily lifestyle, and that q represents the complementary event, then in such cases, survival distributions are used to describe survival behaviour mathematically.
The mixed survival distribution is an important statistical tool in studying the survival behavior of patients or systems. It is used to describe survival times when multiple components influence mortality or failure rates. This distribution is based on the assumption that data may arise from a mixture of different distributions, with each component reflecting the influence of a specific set of factors, such as environmental stressors or biological indicators [4], [6].
Based on the above, and given the stressors experienced by kidney failure patients, let us assume that the probability of the risk function remaining unchanged for kidney failure patients is P, and that the risk function will change with probability q. Then:the mixed hazard function is defined as follows

$$h(t)=p\lambda+(1-p)\lambda t \qquad (1)$$

Where λ represents the scaling parameter. When p=1 we get the constant hazard function for the Exponential distribution, and when p=0, we get the hazard function for the Weibull distribution.
The basic idea is that data do not follow a single distribution, but can result from a mixture of different distributions. The hazard function is used to describe the instantaneous rate of failure or death at time t, and it is a central tool for understanding survival behavior. The survival function represents the

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probability of an individual surviving to a certain point in time and is inversely related to the hazard function. The flexibility of using the mixed distribution lies in its ability to represent complex situations, .such as increasing risk over time or varying responses between individuals In studying patients with chronic kidney failure, the mortality rate can be influenced by multiple factors such as age, biochemical .markers, and daily stressors It is worth noting here that the mixed survival distribution used in this article allows for estimating the actual hazard rate and determining how risks accumulate over time at different levels of influences. It is important to note that the hazard is not constant, but rather varies with .time and the level of influences, which is important for kidney failure patients It also allows for comparison of the performance of this distribution with traditional distributions such as the Weibull distribution, providing a more accurate and realistic model of patient survival behavior. In addition to providing higher modeling accuracy compared to traditional distributions such as Weibull or exponential, the mixed distribution can more realistically represent survival data. Furthermore, it enables understanding of the impact of environmental factors or daily stressors on risk and their accumulation over time, supporting clinical decision-making by providing better information about the timing of medical .interventions or patient follow-up based on predicted risks

Therefore, this article will consider the stressors experienced by kidney failure patients using the mixed survival time distribution to demonstrate the pattern of the risk function under the stressors experienced by kidney failure patients throughout their lives. The importance of this article is highlighted compared to other studies on kidney failure, some of which used classical distributions such as the Exponential and Weibull distributions, and others used complex distributions such as the Weibull-Rayleigh distribution. However, this article sheds light on the mixed survival distribution, considering the stressors experienced by kidney failure patients in their daily lives. Therefore, the importance of the mixed survival time distribution is highlighted by its potential use in describing the survival behavior of patients with kidney failure under the stressors they experience in their daily lives. This research paper is divided into five main sections: The first section provides a general background on the mixed risk function in the presence of stressors experienced by kidney failure patients in their daily lives. The second section discusses related work. The third section presents the functions, derivations used, and the numerical solution algorithm. The fourth section discusses the results obtained. Finally, the fifth section presents the conclusions.

There are several studies that have addressed the hazard function. For example, [2] studied longitudinal multivariate joint models and multivariate survival data, studied the association between failure times, and calculated the marginal and conditional survival functions from the multivariate survival model and applied them to a group of breast cancer patients. [3] Presented an article on the Generalized F-distribution to estimate the survival function and hazard functions. The F-distribution was discussed using standard and constant alternatives using well-known statistical programs in this field, and improved descriptions of the risk of death after clinical AIDS diagnosis at four different periods of HIV treatment. [7] studied the three-parameter general Lindley distribution, which is a composite of the first and second gamma distributions. The properties of the hazard function were determined, and the three parameters were estimated using the maximum likelihood method. An algorithm was also proposed to generate random variables for this distribution. His practical application showed that the generalized Lindley distribution is a strong competitor to lifetime distributions such as gamma, Weibull, and natural logarithm.

The last decade has witnessed a significant increase in studies employing statistical distributions and survival models to analyze data on kidney failure patients, with the aim of arriving at more accurate survival estimates and explaining differences between patients. For example, [5] used the Gompertz distribution to analyze clinical data on kidney failure patients, and the results showed that this distribution provided a good fit to survival data, especially in advanced stages of the disease. In a more recent study, [10] applied competing risk models to data on chronic kidney failure patients and confirmed that these models better reflect the multiple causes of death and failure among patients.

[1] also presented an analysis using semiparametric survival models on dialysis patients, highlighting the role of clinical and biochemical variables in explaining differences between patients, providing more accurate information for predicting survival. On the other hand, [11] developed the Mixture Cure Model to account for the presence of subgroups of patients with long-term survival after kidney transplantation, demonstrating that mixed models are better at representing heterogeneous data than traditional models. In a more recent study, [12] used the Log-Logistic Distribution to analyze data on chronic kidney failure patients in China, and their results demonstrated that this distribution provides an accurate representation of survival data, especially in the early stages of the disease.

Methods

The hazard function is defined as the probability that an individual or system will fail in the interval $(t, t+\Delta t)$, given that it has survived up to time t. In other words, the hazard function represents a conditional probability that describes the instantaneous risk of failure within a very small time interval (t_1, t_2). For a living organism, it reflects the instantaneous mortality rate of an individual who has survived until time t,

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or equivalently, the probability that the survival process will terminate at time t. The hazard function is commonly denoted by h(t) and is mathematically expressed using the following formulas:

$$h(t)=\frac{F(t+\Delta t)-F(t)}{S(t)}=\frac{f(t)}{S(t)}$$

The cumulative distribution function (CDF) of survival is defined as the probability that a patient dies before time t. It is denoted by F(t) and can be expressed mathematically as follows:

$$F(t) = P(T \le t)=\int_0^t f(u)\,du \ ; \ t\ge 0$$

The mixed risk function is defined as in Equation (1), and the mixed survival time distribution is defined using the following relationship:

$$f(t)=h(t)e^{-\int_0^t h(t)dt} \qquad (2)$$

Where h(t) is defined as in Equation (1). By substituting Equation (1) into Equation (2), the distribution of the residence time is as follows:

$$f(t)=[p\lambda+(1-p)\lambda t]e^{-(p\lambda t+0.5\,(1-p)\lambda t^2)} \qquad (3)$$

The parameters of the mixed distribution defined by Equation (3) were estimated by following the following steps:

If we assume that $t_1,t_2,t_3,\ldots\ldots\ldots t_n$ a random sample of survival times has a survival time distribution defined by Equation (3), then the maximum likelihood function is given as follows:

$$L=\prod_{i=1}^{n} f(t_i/\lambda,p)=\lambda^n.\left(\prod_{i=1}^{m}(p+(1-p)t_i)\right)e^{-\lambda\sum_{i=1}^{n}\ (pt_i+0.5\,(1-p)t_i^2)}$$

Taking the logarithm of L, we get

$$\ln L=n\ln\lambda+\sum_{i=1}^{n}\ \ln(p+(1-p)t_i)-\lambda\sum_{i=1}^{n}\left(pt_i+0.5\,(1-p)t_i^2\right)$$

If we assume that the parameter p is known and the parameter λ is unknown, then we will estimate the parameter λ by taking the derivative of the function (lnL) with respect to λ and setting it equal to zero, we obtain the maximum likelihood value of the parameter λ as follows:

$$\hat{\lambda}=\frac{n}{\sum_{i=1}^{n}\left[pt_i+0.5(1-p)t_i^2)\right]}$$

If we assume that the parameter λ is known and the parameter p is unknown, then we will estimate the parameter p by taking the derivative of the function (lnL) with respect to p and setting it equal to zero, we obtain the maximum probability value of the parameter as follows:

$$\sum_{i=1}^{n}\left[\frac{1}{\hat{p}+\frac{t_i}{1-t_i}}\right]-\frac{\lambda}{2}\sum_{i=1}^{n}t_i\,(2-t_i)=0 \qquad (4)$$

Due to the difficulty of solving Equation (4), one of the Newton-Raphson methods will be used to estimate the parameter p, by following the following steps:

let us assume that

$$g(p_0)=\sum_{i=1}^{n}\left[\frac{1}{\hat{p}+\frac{t_i}{1-t_i}}\right]-\frac{\lambda}{2}\sum_{i=1}^{n}t_i\,(2-t_i)=0$$

The steps of the Newton-Raphson method depend on assuming an initial value for the required root using the ordinary least squares (OLS) method, let it be ($p_0=p_j$), and then determining approximate roots for the parameter (p) as in the following equation:

$$\hat{p}=p_{j+1}=p_o-\frac{g(p_o)}{\grave{g}(p_o)} \qquad (5)$$

The initial value $p_o$is substituted in equation (5) and we continue applying Equation (5) and substitutin p_o to get a new value, let it be, p_1, and then we assume that it is the initial value p_1 and we continue until we reach the stage (j+1) when it approaches the required degree of accuracy, and thus we get a value for the parameter (p) such that the difference is as small as possible [9]. Using the stability property of the maximum likelihood estimator, we obtain the estimator of the mixed hazard function h(t) and the estimator of the survival function R(t), as follows:

$$\hat{h}(t_i)=p\hat{\lambda}+(1-p)\hat{\lambda}\ t_i$$

$$\hat{S}(t_i)=e^{-\left(\hat{\lambda}pt_i+\frac{1}{2}(1-p)t_i^2\right)}.$$

Results

This article used a sample of 153 patients with renal failure, 87 males and 66 females. The causes of renal failure varied among several primary factors, including hypertension, which appeared in 43 cases, diabetes, which was recorded in 32 cases, and a combination of diabetes and hypertension in 15 cases. Genetic causes, such as polycystic kidney disease, were also present in 22 cases. Other diverse causes included immune rejection of transplanted kidneys, congenital urinary tract diseases, and other factors. The duration of dialysis for the patients ranged from 6 days to 7,630 days (approximately 250 months).

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The data were fitted to the mixed survival distribution function using the Kolmogorov-Smirnov test to ensure that the research data follow the mixed survival distribution at different values of the parameter λ. The results of (Table 1) showed that the value of the Kolmogorov-Smirnov test statistic is less than the critical value (0.13178) at a significance level of 0.01 at different values of the parameter λ. Therefore, it can be said that the research data follow the mixed survival distribution.

Table 1. Goodness-of-Fit Test Results for Mixed Survival Distribution

#	N	λ_0	P_0	$\hat{\lambda}$	$\hat{P}$	K-S test
1	153	0.003	0.08	0.0267	0.0683	0.0636571
2	153	0.003	0.1	0.0267	0.0683	0.0636571
3	153	0.01	0.2	0.0326	0.27	0.0562024
4	153	0.01	0.3	0.0326	0.27	0.0562024
5	153	0.05	0.4	0.0798	0.807	0.0486749
6	153	0.05	0.5	0.0798	0.807	0.0486749
7	153	0.05	0.6	0.0798	0.807	0.0486749
8	153	0.10	0.7	0.1355	0.96	0.0388659
9	153	0.10	0.8	0.1355	0.96	0.0388659
10	153	0.10	0.9	0.0434	0.96	0.0388659

The maximum likelihood estimators for the mixed survival time distribution were found as shown in (Table 2). From (Figure 1), it was observed that when a kidney failure patient was able, with a percentage ranging from 10% to 90%, to handle the daily life stresses resulting from his illness, the mixed survival time distribution began to increase during the first months of the disease and then decreased with a slope to the right. There is an inverse relationship between the survival function and the survival time, i.e., the longer the survival time, the lower the value of the survival function. Therefore, the older the patient, the lower his survival rate. This matches the behavior of the survival function, as it decreases with time. (Figure 2) shows that the cumulative distribution function for the mixed survival time is an increasing function.

Table 2. Estimate of Mixed Survival Time Distribution

#	N	$\hat{P}$	$\hat{\lambda}$	f(t)
1	153	0.0683	0.0267	$(0.002+0.012t)\ e^{-(0.002t+0.006t^2)}$
2	153	0.0683	0.0267	$(0.002+0.012t)\ e^{-(0.002t+0.006t^2)}$
3	153	0.27	0.0326	$(0.009+0.023t)\ e^{-(0.009t+0.012t^2)}$
4	153	0.27	0.0326	$(0.009+0.023t)\ e^{-(0.009t+0.012t^2)}$
5	153	0.807	0.0798	$(0.064+0.015t)\ e^{-(0.064t+0.008t^2)}$
6	153	0.807	0.0798	$(0.064+0.015t)\ e^{-(0.064t+0.008t^2)}$
7	153	0.807	0.0798	$(0.064+0.015t)\ e^{-(0.064t+0.008t^2)}$
8	153	0.96	0.1355	$(0.130+0.005t)\ e^{-(0.130t+0.002t^2)}$
9	153	0.96	0.1355	$(0.130+0.005t)\ e^{-(0.130t+0.002t^2)}$
10	153	0.96	0.0434	$(0.042+0.002t)\ e^{-(0.042t+0.001t^2)}$

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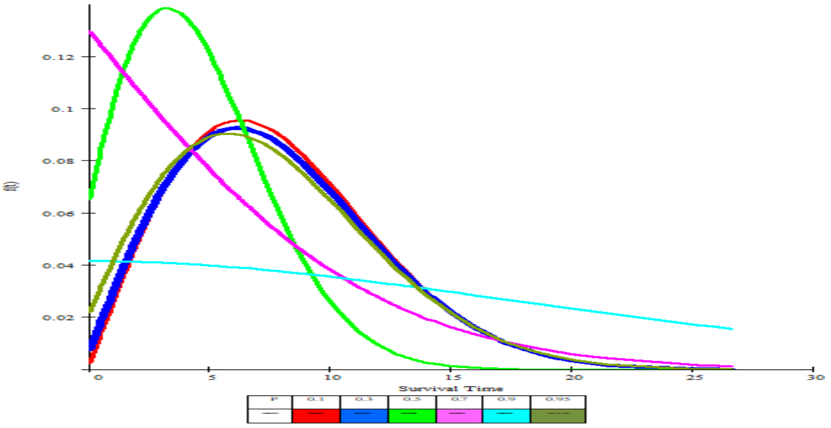

Figure 1. Mixed Survival Time Distribution

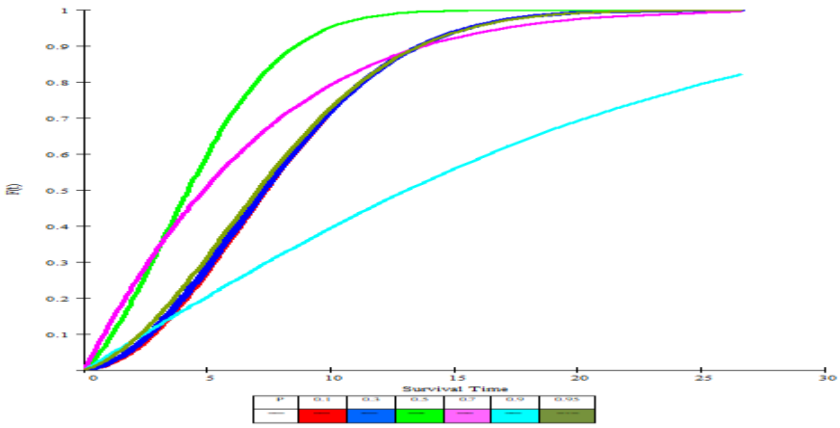

Figure 2. Cumulative Distribution Function Mixed Survival Time Distribution

The maximum likelihood estimators for the mixed survival function were found as shown in (Table 3). From (Figure 3), there is an inverse relationship between the survival function and survival time. That is, the longer the survival time, the lower the survival function value. Therefore, the older the patient, the lower their survival rate. This is consistent with the behavior of the survival function, which is decreasing over time.

Table 3. Estimating The Mixed Survival Function

#	N	$\hat{P}$	$\hat{\lambda}$	S(t)
1	153	0.0683	0.0267	$e^{-(0.002t+0.006t^2)}$
2	153	0.0683	0.0267	$e^{-(0.002t+0.006t^2)}$
3	153	0.27	0.0326	$e^{-(0.009t+0.012t^2)}$
4	153	0.27	0.0326	$e^{-(0.009t+0.012t^2)}$
5	153	0.807	0.0798	$e^{-(0.064t+0.008t^2)}$
6	153	0.807	0.0798	$e^{-(0.064t+0.008t^2)}$
7	153	0.807	0.0798	$e^{-(0.064t+0.008t^2)}$
8	153	0.96	0.1355	$e^{-(0.130t+0.002t^2)}$
9	153	0.96	0.1355	$e^{-(0.130t+0.002t^2)}$
10	153	0.96	0.0434	$e^{-(0.042t+0.001t^2)}$

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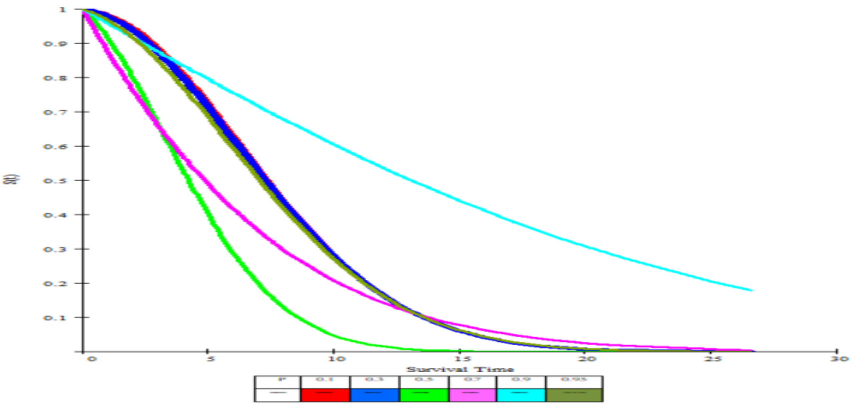

Figure 3. Mixed Survival Function

The maximum likelihood estimators for the mixed hazard function were found, as shown in (Table 4). From (Figure 4), it was observed that when a kidney failure patient is 10% to 99% able to handle the daily stresses of their disease, the hazard function increases with time. This means that the patient is at risk of death during the final weeks of dialysis. This indicates that the hazard rate increases with time, as the hazard function is directly proportional to time. This means that the longer the patient lives, the greater the risk to the patient's life. The curves show that the hazard function starts low at the initial time and gradually increases with the duration of the disease, indicating that the probability of risk exposure to risk increases over time. It also appears that the slope of the lines varies according to the stress levels, as the increase in risk is slow at low stress levels (p=0.1) while it increases significantly at high stress levels (p=0.95). This reflects that stress is an influential factor that contributes to accelerating the risk rate in kidney patients.

Table 4. Estimation of The Mixed Hazard Function

#	N	$\hat{P}$	$\hat{\lambda}$	h(t)
1	153	0.0683	0.0267	(0.002+0.012t)
2	153	0.0683	0.0267	(0.002+0.012t)
3	153	0.27	0.0326	(0.009+0.023t)
4	153	0.27	0.0326	(0.009+0.023t)
5	153	0.807	0.0798	(0.064+0.015t)
6	153	0.807	0.0798	(0.064+0.015t)
7	153	0.807	0.0798	(0.064+0.015t)
8	153	0.96	0.1355	(0.130+0.005t)
9	153	0.96	0.1355	(0.130+0.005t)
10	153	0.96	0.0434	(0.042+0.002t)

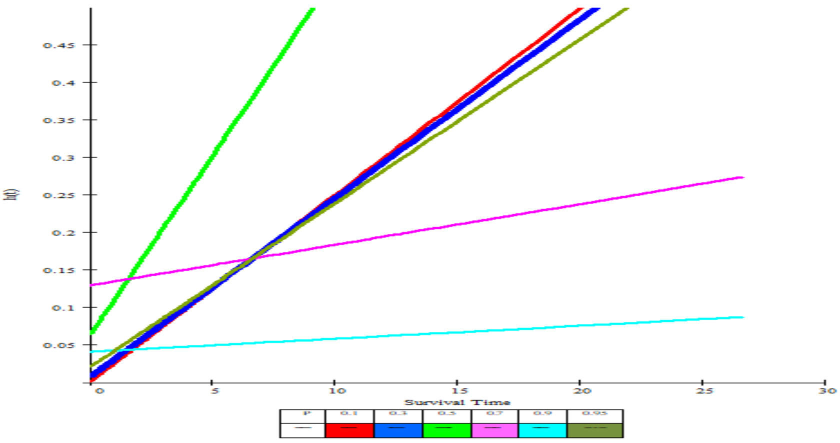

Figure 4. Mixed Hazard Function

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Maximum likelihood estimators for the mixed cumulative hazard function were found, as shown in (Table 5). (Figure 5) shows a direct relationship between the mixed cumulative hazard function and survival time. This means that the longer the survival time, the higher the value of the mixed cumulative hazard function. Therefore, the older the patient, the greater the risk to their life and the greater their risk of death. It appears that the function starts low and then increases exponentially over time, reflecting that the accumulated probability of risk exposure to risk increases with survival time. The curves also differ according to the levels of stress, with the cumulative hazard function rising faster at high levels (p=0.95, p=0.9), while the increase is slower at low levels (p=0.1, p=0.3). This suggests that the severity of stress clearly contributes to accelerating the accumulation of risk in kidney patients.

Table 5. Estimate of The Mixed Cumulative Hazard Function

#	N	$\widehat{P}$	$\hat{\lambda}$	H(t)
1	153	0.0683	0.0267	$(0.002t+0.006t^2)$
2	153	0.0683	0.0267	$(0.002t+0.006t^2)$
3	153	0.27	0.0326	$(0.009t+0.012t^2)$
4	153	0.27	0.0326	$(0.009t+0.012t^2)$
5	153	0.807	0.0798	$(0.064t+0.008t^2)$
6	153	0.807	0.0798	$(0.064t+0.008t^2)$
7	153	0.807	0.0798	$(0.064t+0.008t^2)$
8	153	0.96	0.1355	$(0.130t+0.002t^2)$
9	153	0.96	0.1355	$(0.130t+0.002t^2)$
10	153	0.96	0.0434	$(0.042t+0.001t^2)$

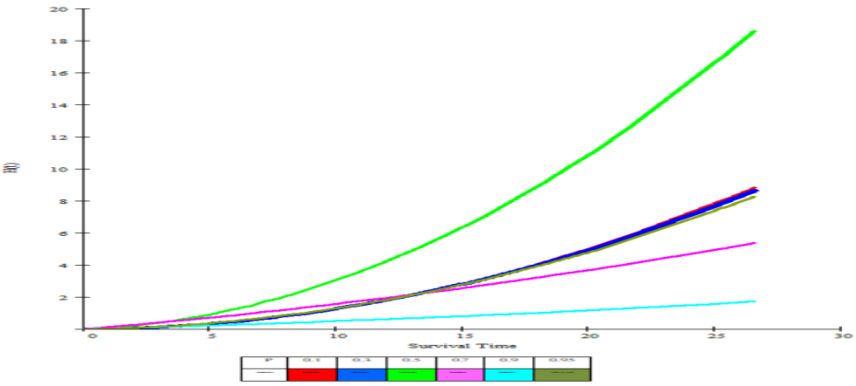

Figure 5. Mixed Cumulative Hazard Function

The parameters of the Pareto, Pareto (2P), Weibull, Cauchy, Dagum, Exponential (2P), and Gamma distributions were estimated using the maximum likelihood method using the Easy-Fit program. These distributions were applied to real-world data on survival times for patients with kidney failure. The data were fitted to the Pareto, Pareto (2P), Weibull, Cauchy, Dagum, Exponential (2P), and Gamma distributions. Comparison criteria were also used to compare the distributions to select the best distribution representing the article data. The results are shown in (Table 6). The data were fitted to some classical distributions (Pareto, Pareto (2P), Weibull, Cauchy, Dagum, Exponential (2P), Gamma) using the Kolmogorov-Smirnov test to ensure that the research data follow these distributions. Based on the results of (Table 6), it was found that the value of the Kolmogorov-Smirnov test statistic is less than the critical value (0.13178) at a significance level of 0.01. Therefore, it can be said that the research data follow classical distributions (Pareto (2P), Weibull, Dagum, Exponential (2P), Gamma), and do not follow the classical distributions (Pareto, Cauchy). Based on the results of the Akaike Information Criterion (AIC) and Modified Akaike Information Criterion (MAIC) shown in (Table 6), it was found that the two-parameter Weibull distribution best represents the research data, as its comparison criteria values were lower than those of other distributions.

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Table 6. Estimating the Parameters of Some Classical Distributions, Goodness-of-Fit Test Results and Comparison Criteria Between Classical Distributions

#	Distribution	Parameters	KS test	Decision	AIC	MAIC
1	Cauchy	$\sigma = 2.24, \mu = 0.3$	0.21009	Reject	-	-
2	Dagum	$\kappa = 0.44, \alpha = 1.99, \beta = 7.34$	0.04811	Accept	863.641	863.694
3	Exponential	$\lambda = 1.17$	0.08112	Accept	857.211	857.237
4	Exponential (2P)	$\lambda = 0.17, \gamma = 0.02$	0.08207	Accept	858.211	858.181
5	Gamma	$\alpha = 0.83, \beta = 7.27$	0.04796	Accept	858.141	858.100
6	Pareto	$\alpha = 0.20, \beta = 0.02$	0.37727	Reject	-	-
7	Pareto-2P	$\alpha = 7.36, \beta = 38.4$	0.04852	Accept	857.296	857.366
8	Weibull	$\alpha = 0.92, \beta = 5.63$	0.04645	Accept	857.166	857.206

The maximum likelihood estimators of the hazard function were found. The results are shown in (Table 7). and through (Figure 6), it was shown that the hazard function increases with time. That is, the hazard function begins to increase during the first weeks of dialysis and then begins to rise until it reaches its maximum value. This means that the patient is at risk of death during the last weeks of dialysis. This indicates that the risk rate increases with time, as this function is directly proportional to time. This indicates that the longer the survival time of the disease, the greater the risk to the patient's life. There is a direct relationship between the mixed cumulative hazard function and survival time. That is, the longer the survival time, the higher the value of the mixed cumulative hazard function. Therefore, the older the patient, the greater the risk to the patient's life and the risk of death.

Table 7. Estimation of The Hazard Function and Cumulative Hazard Function for the Weibull Distribution

functions	Estimated functions
Hazard function	$0.16 \left(\frac{t}{5.6}\right)^{-0.08}$
Cumulative hazard function	$\left(\frac{t}{5.6}\right)^{0.9}$

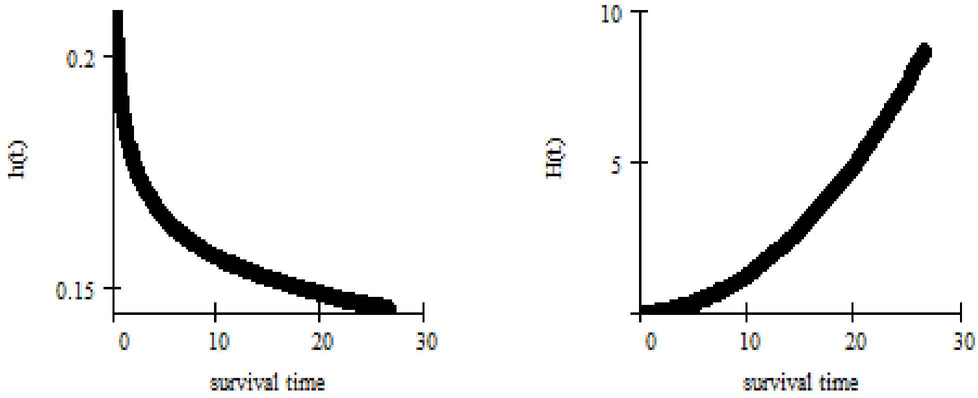

Figure 6. The Weibull Distribution Hazard Function and The Cumulative Hazard Function

The comparison between the mixed hazard function and the Weibull distribution hazard function was conducted using mean square error. The results of (Tables 3-17, 3-18, and 3-19) reveal that the mean square error for the mixed hazard function is less than the mean square error for the Weibull distribution. This indicates that the mixed distribution is a better representation of the research data than the Weibull distribution.

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Tables 8. Mean Squared Error for The Mixed Hazard Function

#	N	0λ	$_0P$	$\hat{P}$	$\hat{\lambda}$	MSE(h(t))
1	153	0.003	0.08	0.0683	0.0267	7.6299×10^{-26}
2	153	0.003	0.1	0.0683	0.0267	7.6054×10^{-26}
3	153	0.01	0.2	0.27	0.0326	7.6544×10^{-26}
4	153	0.01	0.3	0.27	0.0326	7.6238×10^{-26}
5	153	0.05	0.4	0.807	0.0798	1.7112×10^{-21}
6	153	0.05	0.5	0.807	0.0798	1.7113×10^{-21}
7	153	0.05	0.6	0.807	0.0798	1.7113×10^{-21}
8	153	0.1	0.7	0.96	0.1355	4.5723×10^{-19}
9	153	0.1	0.8	0.96	0.1355	4.5724×10^{-19}
10	153	0.1	0.9	0.96	0.0434	4.5724×10^{-19}

Table 9. Mean Square Error of The Weibull Distribution

Function	MSE(h(t))
$h(t)=0.16\left(\frac{t}{5.6}\right)^{-0.08}$	2.000×10^{-7}

Discussion

This article used a sample of 153 patients with kidney failure. The data were fitted to a mixed survival distribution. Estimating the mixed survival time distribution revealed that when a kidney failure patient was 10% to 90% able to cope with the daily stresses of their disease. The combined hazard function was found to be directly proportional to time, and the stressors experienced by the patient significantly accelerated the risk rate in kidney patients. This means that the longer the patient lived, the greater the risk to their life. The curves of the mixed hazard function showed that the slope of the lines varied according to the stress level, with the increase in risk being slow at low stress levels and increasing significantly at high stress levels.

Estimating the mixed cumulative hazard function, it was found that the severity of the stressors experienced by the patient clearly contributed to the acceleration of the risk accumulation in kidney patients. That is, the longer the survival time, the higher the value of the mixed cumulative hazard function. Therefore, the older the patient, the greater the risk to their life and the greater the risk of death. The mixed cumulative hazard function curves showed that the slope of the lines varied according to stress levels, with the cumulative hazard function rising faster at high levels and slower at low levels. To demonstrate that the mixed distribution outperformed classical distributions, the data were fitted to classical distributions. The results of the comparison criteria indicated that the Weibull distribution was the optimal distribution for the data. However, this distribution was not superior to the mixed distribution, as demonstrated by comparing the mixed hazard function and the Weibull hazard function using the mean square error.

Accordingly, this article presents a mixed distribution that illustrates the behavior of the risk function for kidney failure patients, taking into account the stressors of illness and the psychological pressures that kidney failure patients experience in their daily lives. It is worth noting that most studies have not used the mixed distribution in medical studies, and not all studies have examined the association between the mixed distribution and the stressors that kidney failure patients experience. We hope that this article will be a starting point for new research that goes beyond the use of distributions alone, but also takes into account the psychological aspects of patients.

Conclusion

This article discusses the use of the mixed survival distribution to describe the survival behavior of kidney failure patients, taking into account the stressors they face in their daily lives. It analyzes the associated hazard function and compares it to classical distributions, particularly the Weibull distribution. The sample included data from 153 patients at the Kidney Services Center in Al-Khums. The results showed that the mixed distribution provides a better fit to real-world data than the Weibull distribution, as the hazard function increased over time, reflecting a higher probability of death, while the survival function decreased. The analyses also showed that daily stress levels are an influencing factor that accelerates and accumulates the hazard rate, with the escalation being gradual at low stress levels and steep at high stress levels. The study concludes that the mixed distribution is more accurate and realistic in representing survival behavior and recommends its use in similar studies and conducting additional analyses on different types of data to test the generality of the results.

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